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The Gluon Propagator in Minkowski and Euclidean Space: Role of an A^2 Condensate

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Abstract

In a recent work we described the form taken by the Minkowski-space gluon propagator in the presence of an $< A_\mu^a A_a^\mu >$ condensate. This condensate has been seen to play an important role in calculations which make use of the operator product expansion. The latter work has led to the publication of a large number of papers which discuss how the $< A_\mu^a A_a^\mu >$ condensate could play a role in a gauge-invariant formulation. In the present work we discuss the properties of both the Euclidean-space and Minkowski-space gluon propagator. In the case of the Euclidean-space propagator we can make contact with the results of QCD lattice calculations of the propagator in the Landau gauge. With an appropriate choice of normalization constants, we present a unified representation of the gluon propagator that describes both the Minkowski-space and Euclidean-space dynamics in which the $< A_\mu^a A_a^\mu >$ condensate plays an important role.

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I. INTRODUCTION

Recently, studies making use of the operator product expansion (OPE) have provided evidence for the importance of the condensate $< A_\mu^a A_a^\mu >$ [1-3]. (There is a suggestion that such a condensate may be related to the presence of instantons in the vacuum [4].) The importance of that condensate raises the question of gauge invariance and there are now a large number of papers that address that and related issues [5-19]. We will not attempt to review that large body of literature, but will consider how the presence of an $< A_\mu^a A_a^\mu >$ condensate affects the form of the gluon propagator. We may mention the work of Kondo [7] who was responsible for introducing a BRST-invariant condensate of dimension two,

$$\mathcal{Q} = \frac{1}{\Omega} < \int d^4x \, \mathrm{Tr}\left(\frac{1}{2}A_\mu(x)A_\mu(x) - \alpha i c(x) \cdot \bar{c}(x)\right) >, \tag{1.1}$$

where $c(x)$ and $\bar{c}(x)$ are Faddeev-Popov ghosts, α is the gauge-fixing parameter and Ω is the integration volume. Kondo points out that Ω reduces to A^2_{min} in the Landau gauge, $\alpha = 0$. The minimum value of the integrated squared potential is A^2_{min}, which has a definite physical meaning [7].

In a recent work we considered the Minkowski-space gluon propagator in the presence of an A^2 condensate [20]. We found that the propagator has no on-mass-shell poles, so that the gluon was a nonpropagating mode in the presence of the vacuum condensate [21]. The form we obtained for the propagator was

$$D^{\mu\nu}(k) = \left(g^{\mu\nu} - \frac{k^\mu k^\nu}{k^2}\right) D(k) \tag{1.2}$$

with

$$D(k) = \frac{Z_1}{k^2 - m^2 + \frac{4}{3}\frac{k^2 m^2}{k^2 - m^2}}. \tag{1.3}$$

Here Z_1 is a normalization parameter which we put equal to 3.82 so that we may obtain a continuous representation as we pass from Minkowski to Euclidean space. In Fig. 1 we show $D(k)$ with $m^2 = 0.25$ GeV^2. (We remark that $D(k) = 0$ when $k^2 = m^2$, $D(k) = -Z_1/m^2$ at $k^2 = 0$, and $D(k) \to Z_1/k^2$ for large k^2.) If we choce $Z_1 = 15.28m^2 = 3.82$ our result for the propagator will be continuous at $k^2 = 0$ when we consider both the Euclidean-space and Minkowski-space propagators.

The organization of our work is as follows. In Section II we discuss the Euclidean-space gluon propagator as obtained in a lattice simulation of QCD in the Landau gauge and in

the absence of quark degrees of freedom [22]. (We also record in the Appendix a number of semi-phenomenological analytic forms which are meant to represent the Euclidean-space propagator.) In Section III we summarize the results of our analysis and provide some additional discussion.

II. QCD LATTICE CALCULATIONS AND PHENOMENOLOGICAL FORMS FOR THE EUCLIDEAN-SPACE GLUON PROPAGATOR

Results for the gluon propagator obtained in a lattice simulation of QCD are given in Ref. [22]. In that work the authors also record several phenomenological forms. We reproduce these forms in the Appendix for ease of reference. Of these various forms we will make use of model A of Ref. [22] which has the form

$$D^L(k^2) = Z\left[\frac{AM^{2\alpha}}{(k^2+M^2)^{1+\alpha}} + \frac{1}{k^2+M^2}L(k^2,M)\right], \tag{2.1}$$

with

$$L(k^2,M) \equiv \left[\frac{1}{2}\ln(k^2+M^2)(k^{-2}+M^{-2})\right]^{-d_D}, \tag{2.2}$$

and $d_D = 13/22$. The parameters used in Ref. [22] to provide a very good fit to the QCD lattice data are

$$Z = 2.01^{+4}_{-5}, \tag{2.3}$$

$$A = 9.84^{+10}_{-86}, \tag{2.4}$$

$$M = 0.54^{+5}_{-5}, \tag{2.5}$$

and

$$\alpha = 2.17^{+4}_{-19}. \tag{2.6}$$

Note that M in GeV units is 1.018 GeV. Rather than work with the lattice data we will use Eqs. (2.1)-(2.6) when we compare our results with the lattice data. In Fig. 2 we show $k^2 D^L(k)$ of Eq. (2.1) and in Fig. 3 we show $D^L(k)$. These functions are represented by the solid lines in Figs. 2 and 3. Note that Eq. (1.3) may be written in Euclidean space as

$$D_E(k) = -\frac{Z_1}{k_E^2 + m^2 - \frac{4}{3}\frac{k_E^2 m^2}{k_E^2+m^2}}. \tag{2.7}$$

This form is useful for $k_E^2 < 1$ GeV^2 and we therefore consider various phenomenological forms which may be used to extend Eq. (2.7) so that we may attempt to fit the lattice result over a broader momentum range. To that end, we make use of Ref. [23]. The authors of that work define the Landau gauge gluon propagator as

$$< A_\mu^a(k) A_\nu^a(k') >= V\delta(k+k')\delta^{ab}\left(\delta_{\mu\nu} - \frac{k_\mu k_\nu}{k^2}\right)\frac{Z(k^2)}{k^2}, \tag{2.8}$$

with

$$Z(k^2) = \omega\left(\frac{k^2}{\Lambda_{QCD}^2 + k^2}\right)^{2\kappa}(\alpha(k^2))^{-\gamma}, \tag{2.9}$$

and $\gamma = -13/22$. (We do not ascribe any particular significance to Eq. (2.9). We use Eq. (2.9) as a phenomenological form which could be replaced by a form which provides a better fit to the data within the context of our model at some future time. We believe Eq. (2.9) is useful, since it is a simple matter to remove the first term of that equation and introduce a propagator that has the small k^2 behavior of our model.)

The authors of Ref. [23] introduce two choices for $\alpha(k^2)$ of Eq. (2.9). We use their form for $\alpha_2(k^2)$:

$$\alpha_2(k^2) = \frac{\alpha(0)}{\ln\left[e + a_1\left(\frac{k^2}{\Lambda_{QCD}^2}\right)^{a_2}\right]}. \tag{2.10}$$

In their analysis they put $\kappa = 0.5314$, $\Lambda_{QCD} = 354$ MeV, $\alpha(0) = 2.74$, $a_1 = 0.0065$ and $a_2 = 2.40$. (Here, we have not recorded the uncertainties in these values which are given in Table 2 of Ref. [23].) As we proceed, we will change these values somewhat. As a first step we remove the first factor in Eq. (2.9) and write

$$Z(k^2) = Z_2(\alpha_2(k^2))^{-\gamma}. \tag{2.11}$$

We now use $a_1 = 0.0080$ and $a_2 = 2.10$ rather than the values given above. In Fig. 4 we show $(\alpha_2(k))^{13/22}$ as a function of k, using our modified values of a_1 and a_2.

We now define

$$D_E(k_E) = -\frac{Z_2(\alpha(k^2))^{-\gamma}}{k_E^2 + m^2 - \frac{4}{3}\frac{k_E^2 m^2}{k_E^2 + m^2}}. \tag{2.12}$$

The function $-k^2 D_E(k_E)$ is shown in Fig. 2 as a dotted line. In this calculation we have put $Z_2 = 2.11$. We find a good representation of the lattice result for $k_E < 2$ GeV,

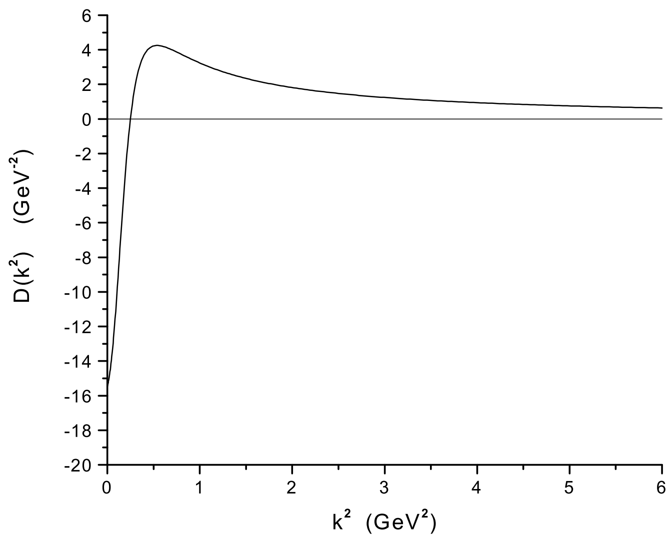

FIG. 1: The function $D(k^2)$ of Eq. (1.3) is shown in Minkowski space. The value for large k^2 is given by Z_1/k^2 with $Z_1 = 3.87$. Here $m = 0.50$ GeV.

In Fig. 3 we compare $D_E(k_E)$ with the result of the lattice calculation which is represented by the solid line. In Fig. 5 we combine our results in Minkowski and Euclidean space and show the values of $k^2D(k^2)$ for both positive and negative k^2 values. For positive k^2 we use $D(k)$ of Eq. (1.3) and for negative values of k^2 we use $D_E(k_E^2)$ of Eq. (2.12). Equality of these functions at $k^2 = 0$ implies $Z_1 = Z_2(\alpha(0))^{13/22}$, or $Z_1 = 1.81Z_2$. (In our work we have used $Z_1 = 3.82$ and $Z_2 = 2.11$. See Eqs. (1.3) and (2.12).) In Fig. 6 we show $D(k^2)$ rather than $k^2D(k^2)$, which was shown in Fig. 5.

III. DISCUSSION AND CONCLUSIONS

In this work we have provided a representation of the gluon propagator in both Euclidean and Minkowski space. The Minkowski-space propagator has only complex poles and that implies that the gluon is a nonpropagating mode in the QCD vacuum. Our analysis takes into account the important condensate $< A_a^\mu A_\mu^a >$ which is responsible for mass generation for the gluon. Our work has some relation to that of Cornwall [24] who obtained a gluon mass of 500 ± 200 MeV in his analysis. Cornwall also suggested that "quark confinement arises from a vertex condensate supported by a mass gap."

In recent work Gracey obtained a pole mass of the gluon of $2.13\Lambda_{\overline{MS}}$ in a two-loop renormalization scheme [25]. If we put $\Lambda_{\overline{MS}} = 250$ MeV, the mass obtained at two-loop order in Ref. [25] is 532 MeV, which is close to the value of 500 MeV used in the present

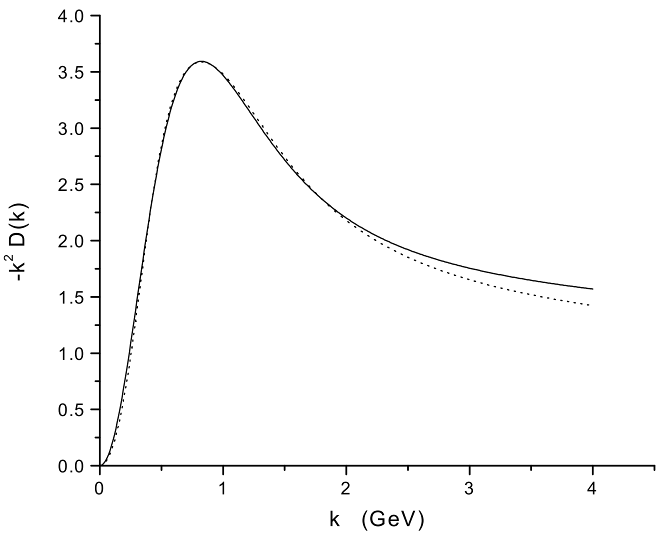

FIG. 2: The function $-k_E^2 D_E(k)$ is shown. The solid line represents the QCD lattice data, while the dotted line represents $-k_E^2 D_E(k)$ in the case that $D_E(k)$ is given in Eq. (2.12).

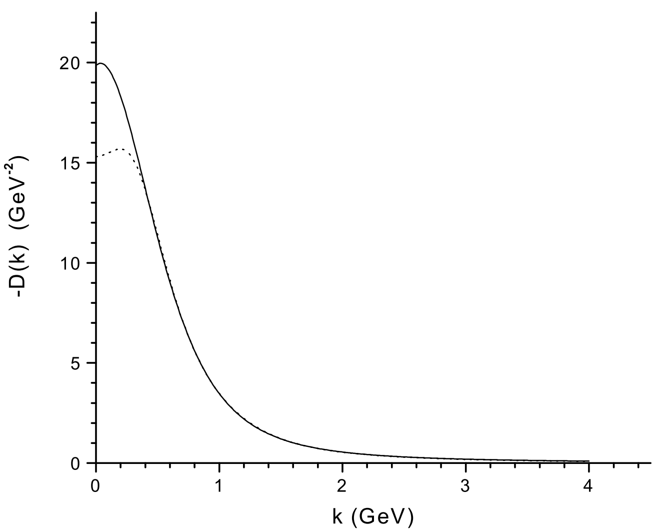

FIG. 3: The function $-D_E(k)$ is shown. The solid line represents the QCD lattice data, while the dotted line represents $-D_E(k)$ of Eq. (2.12). [See Fig. 2.]

work. (We remark that in Ref. [21] we obtained a gluon mass of 530 MeV, if we made use of Eq. (3.18) of that reference, which includes the effect of including various exchange terms in our analysis of the relevant matrix elements.)

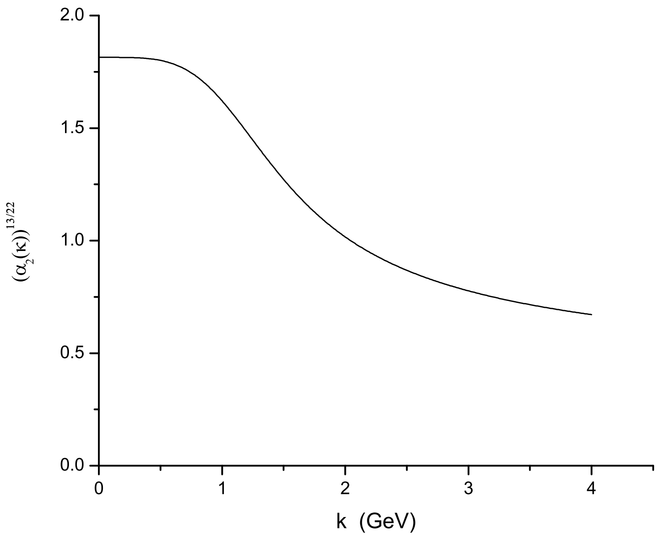

FIG. 4: The function $(\alpha_2(k))^{13/22}$ is shown. [See Eq. (2.10).] Note that $(\alpha_2(0))^{13/22} = 1.81$.

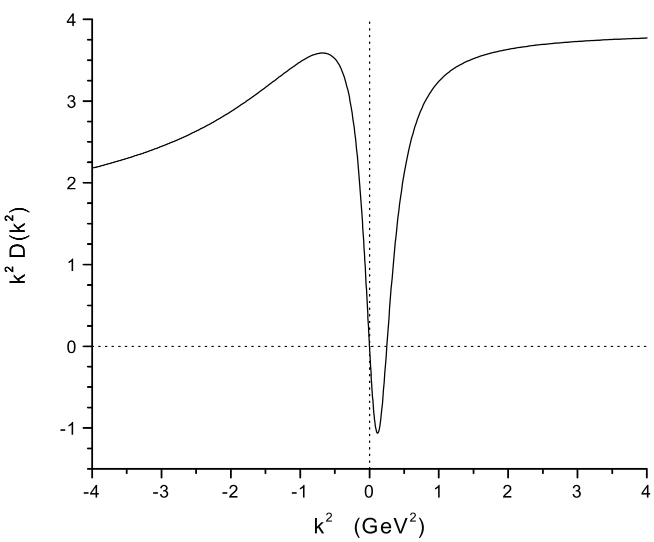

FIG. 5: For $k^2 > 0$ the solid line represents $k^2 D(k^2)$ with $D(k^2)$ given by Eq. (1.3). Here, $Z_1 = 3.82$. For $k^2 < 0$ we show $k^2 D_E(k^2)$, where $D_E(k_E^2)$ is given by Eq. (2.12) with $Z_2 = 2.11$.

APPENDIX A

For ease of reference we record various semi-phenomenological forms which are meant to represent the Euclidean-space gluon propagator.

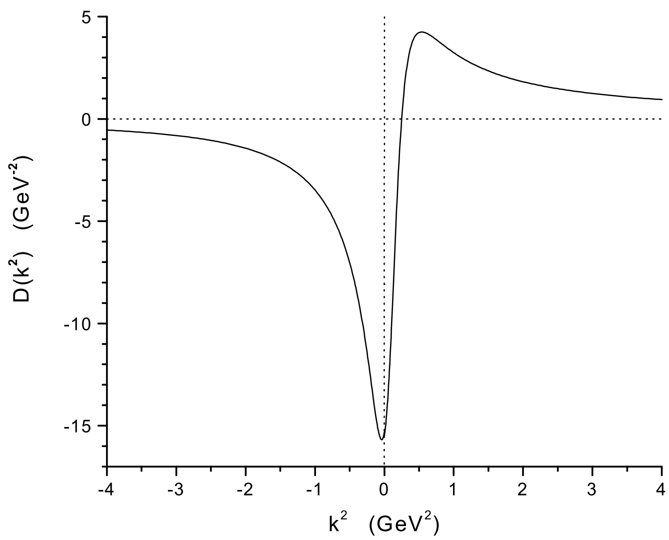

FIG. 6: Same as Fig. 5 except that $D(k^2)$ is shown.

Gribov [26]:

$$D^L(k^2) = \frac{Zk^2}{k^4 + M^4} L(k^2, M). \tag{A1}$$

Stingl [27]:

$$D^L(k^2) = \frac{Zk^2}{k^4 + 2A^2k^2 + M^4} L(k^2, M). \tag{A2}$$

Marenzoni et al. [28]:

$$D^L(k^2) = \frac{Z}{(k^2)^{1+\alpha} + M^2}. \tag{A3}$$

Cornwall I [24]:

$$D^L(k^2) = Z \left[[k^2 + M^2(k^2)] \ln\left(\frac{k^2 + 4M^2(k^2)}{\Lambda^2} \right) \right]^{-1}, \tag{A4}$$

where

$$M(k^2) = M \left[\frac{\ln\left(\frac{k^2+4M^2}{\Lambda^2}\right)}{\ln\left(\frac{4M^2}{\Lambda^2}\right)} \right]^{-6/11}. \tag{A5}$$

Cornwall II [29]:

$$D^L(k^2) = Z \left[[k^2 + M^2] \ln\left(\frac{k^2 + 4M^2}{\Lambda^2} \right) \right]^{-1}. \tag{A6}$$

Cornwall III [29]:

$$D^L(k^2) = \frac{Z}{k^2 + Ak^2 \ln\left(\frac{k^2}{M^2}\right) + M^2}. \tag{A7}$$

Model A [22]:

$$D^L(k^2) = Z\left[\frac{AM^{2\alpha}}{(k^2+M^2)^{1+\alpha}} + \frac{1}{k^2+M^2}L(k^2,M)\right]. \tag{A8}$$

The parameters for model A are given in Eqs. (2.3)-(2.6).

Model B [22]:

$$D^L(k^2) = Z\left[\frac{AM^{2\alpha}}{(k^2)^{1+\alpha}+(M^2)^{1+\alpha}} + \frac{1}{k^2+M^2}L(k^2,M)\right]. \tag{A9}$$

Model C [22]:

$$D^L(k^2) = Z\left[\frac{A}{M^2}e^{-(k^2/M^2)^\alpha} + \frac{1}{k^2+M^2}L(k^2,M)\right]. \tag{A10}$$

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